On Gauge Invariance and Spontaneous Symmetry Breaking *

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Abstract

We show how the widely used concept of spontaneous symmetry breaking can be explained in causal perturbation theory by introducing a perturbative version of quantum gauge invariance. Perturbative gauge invariance, formulated exclusively by means of asymptotic fields, is discussed for the simple example of Abelian $U(1)$ gauge theory (Abelian Higgs model). Our findings are relevant for the electroweak theory, as pointed out elsewhere.

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1 Introduction

It is quite a common assumption that scalar QED with massive photons is not a gauge theory *in the usual sense*, because the introduction of a mass term in the Lagrangean for the gauge field violates the *classical gauge invariance* of the theory. Therefore, a 'Higgs' field with non-vanishing vacuum expectation value is usually coupled to the photon which then acquires a mass [1]. Proceeding in this way, the local $U(1)$ invariance is not absent, but 'hidden'.

It is the aim of this paper to demonstrate how massive gauge theories can be described in the framework of causal perturbation theory [2] by means of a perturbative version of quantum gauge invariance (25). Perturbative gauge invariance has the advantage that it provides a powerful tool for the actual construction of the theory. We will demonstrate this for the Abelian Higgs model in Sect. 4. [1] Starting from a cubic coupling $\sim A_\mu A^\mu \phi$, gauge invariance of first order demands the introduction of scalar ghost fields $u, \tilde{u}$ and of an additional unphysical scalar field Φ and fixes most of the cubic couplings. Then, gauge invariance to second order determines the remaining cubic couplings and requires additional quartic ones. One has to go to third order to fix the quartic couplings completely. The resulting couplings contain the Higgs potential which, however, comes out as a quartic polynomial in the original asymptotic scalar field ϕ with vanishing vacuum expectation value $\langle \phi \rangle = 0$. That means, gauge invariance leads us directly to the final theory 'after spontaneous symmetry breaking'. Although we can see the symmetry breaking in the double-well potential at the end, it plays no direct role in the construction: perturbative gauge invariance alone does the job.

The method beautifully works in the more complicated situation of the electroweak theory, as pointed out in detail elsewhere [9,10].

2 Gauge Invariance for Massive Gauge Fields

[1] We would like to thank Bert Schroer for posing this problem

2.1 Causal Perturbation Theory

Our work is best done in the framework of causal perturbation theory, which has its roots in a classical paper by Epstein and Glaser [2]. In this approach the S-matrix is constructed inductively order by order in the form

$$S(g) = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \, T_n(x_1, \ldots x_n) g(x_1) \ldots g(x_n), \tag{1}$$

where $g(x)$ is a tempered test function that switches the interaction. The first order (e.g. for QED)

$$T_1(x) = ie : \bar{\Psi}(x)\gamma^{\mu}\Psi(x) : A_{\mu}(x) \tag{2}$$

must be given in terms of the asymptotic free fields. It is a striking property of the causal approach that *no ultraviolet divergences* appear, i.e. the T_n's are finite and well defined up to finite normalization terms. The adiabatic limit $g(x) \to 1$ has been shown to exist in purely massive theories in each order [2].

The crucial point in the causal formulation of perturbation theory is that the usual formal definition of the T_n via simple time-ordering

$$T_n(x_1, ...x_n) = T\{T_1(x_1) \cdot ...T_1(x_n)\} \tag{3}$$

$$\equiv \sum_{\Pi} \Theta(x^o_{\Pi_1} - x^o_{\Pi_2}) \cdot ...\Theta(x^o_{\Pi_{n-1}} - x^o_{\Pi_n}) T_1(x_{\Pi_1}) \cdot ...T_1(x_{\Pi_n}), \tag{4}$$

where the sum runs over all $n!$ permutations, contains ultraviolet divergences, therefore there must be an error in the derivation. Epstein and Glaser proceed more carefully and introduce the following n-point distributions:

$$A'_n(x_1, \ldots x_n) = \sum_{P_2} \tilde{T}_{n_1}(X) T_{n-n_1}(Y, x_n), \tag{5}$$

$$R'_n(x_1, \ldots x_n) = \sum_{P_2} T_{n-n_1}(Y, x_n) \tilde{T}_{n_1}(X), \tag{6}$$

where the sums run over all partitions

$$P_2 : \quad \{x_1, \ldots x_{n-1}\} = X \cup Y, \quad X \neq \emptyset \tag{7}$$

into disjoint subsets with $|X| = n_1$, $|Y| \leq n-2$. Assuming by induction that $T_1, ...T_{n-1}$ are known, then A'_n and R'_n can be calculated. One also introduces

$$D_n(x_1, \ldots x_n) = R'_n - A'_n. \tag{8}$$

If the sums are extended over all partitions P_2^0, including the empty set $X = \emptyset$, we obtain the distributions

$$A_n(x_1, \ldots x_n) = \sum_{P_2^0} \tilde{T}_{n_1}(X) T_{n-n_1}(Y, x_n) = \tag{9}$$

$$= A'_n + T_n(x_1, \ldots x_n), \tag{10}$$

$$R_n(x_1, \ldots x_n) = \sum_{P_2^0} T_{n-n_1}(Y, x_n) \tilde{T}_{n_1}(X) = \tag{11}$$

$$= R'_n + T_n(x_1, \ldots x_n). \tag{12}$$

These two distributions are not known by the induction assumption because they contain the unknown T_n. Only the difference

$$D_n = R'_n - A'_n = R_n - A_n \tag{13}$$

is known. We stress the fact that all products of distributions in here are well-defined because the arguments are disjoint sets of points so that the products are tensor products of distributions.

One can determine R_n or A_n separately by investigating the support properties of the various distributions. Causality of the S-matrix requires that R_n is a retarded and A_n an advanced distribution [2,3]

$$\mathrm{supp}\, R_n \subseteq \bar{\Gamma}^+_{n-1}(x_n), \quad \mathrm{supp}\, A_n \subseteq \bar{\Gamma}^-_{n-1}(x_n), \tag{14}$$

with

$$\bar{\Gamma}^\pm_{n-1}(x) \equiv \{(x_1, \ldots x_{n-1}) \mid x_j \in \bar{V}^\pm(x), \forall j = 1, \ldots n-1\},$$

$$\bar{V}^\pm(x) = \{y \mid (y-x)^2 \geq 0,\ \pm(y^0 - x^0) \geq 0\}. \tag{15}$$

Hence, by splitting of the causal distribution (13) one gets R_n (and A_n), and T_n then follows from (10) (or (12)). The T_n's so obtained are well-defined time-ordered products. Local terms

with support $(x_1 = ... = x_n)$, originating from a certain ambiguity in the splitting procedure, might spoil the symmetry of the T_n's in $x_1, ...x_n$, but this minor problem can be removed by subsequent symmetrization.

To carry out the splitting process, we write (13) in normally ordered form and split the numerical distributions $d_n^k(x)$, where $x = (x_1 - x_n, ..., x_{n-1} - x_n)$

$$D_n(x_1, ...x_n) = \sum_{\mathcal{O}} d_n^{\mathcal{O}}(x_1 - x_n, ...x_{n-1} - x_n) : \mathcal{O}(x_1, ...x_n) : . \tag{16}$$

$: \mathcal{O} :$ is a normally ordered product of external field operators (Wick monomial). It is a consequence of translation invariance that $d_n^{\mathcal{O}}(x)$ depends only on relative coordinates.

The only nontrivial step in the construction of well-defined time-ordered products is the splitting of a numerical distribution d with support in $\bar{\Gamma}^+ \cup \bar{\Gamma}^-$ into a distribution r with support in $\bar{\Gamma}^+$ and a distribution a with support in $\bar{\Gamma}^-$. In causal perturbation theory the usual formal time-ordered products with subsequent renormalization are replaced by this conceptually simple and mathematically well-defined procedure. In fact the problem of distribution splitting was already solved in a general framework by the mathematician Malgrange in 1960 [4]. Epstein and Glaser used his general result for the special case of relativistic quantum field theory [2]. A simple solution for the splitting problem can be found in [3].

2.2 Gauge Invariance for Massive QED

Since the above construction of the perturbative S-matrix uses only the asymptotic free fields, we are looking for a formulation of quantum gauge invariance in terms of these fields.

We discuss first the simple case of quantum electrodynamics with massive photons. Let

$$Q \stackrel{def}{=} \int d^3x (\partial_\mu A^\mu(x) + m\Phi(x)) \overleftrightarrow{\partial_0} u_a(x) \tag{17}$$

be the generator of (free) gauge transformations, called gauge charge for brevity. A_μ is the gauge potential in the Feynman gauge, u, $\tilde{u}$ are fermionic ghost fields and Φ a neutral scalar, satisfying

the well-known commutation relations

$$[A_\mu^{(\pm)}(x), A_\nu^{(\mp)}(y)] = ig^{\mu\nu} D_m^{(\mp)}(x-y), \tag{18}$$

$$\{u^{(\pm)}(x), \tilde{u}^{(\mp)}(y)\} = -iD_m^{(\mp)}(x-y), \tag{19}$$

$$[\Phi^{(\pm)}(x), \Phi^{(\mp)}(y)] = -iD_m^{(\mp)}(x-y) \tag{20}$$

and all other {anti-}commutators vanish. All these fields fulfil the Klein-Gordon equation with the same mass m. In order to see how the infinitesimal gauge tranformation acts on the free fields, we calculate the (anti-)commutators [9]

$$[Q, A_\mu] = i\partial_\mu u \quad , \quad [Q, \Phi] = imu \quad , \tag{21}$$

$$\{Q, u\} = 0 \quad , \quad \{Q, \tilde{u}\} = -i\partial_\mu A^\mu - im\Phi \qquad , \quad [Q, \Psi] = 0. \tag{22}$$

Then we have

$$[Q, T_1(x)] = -e : \bar{\Psi}\gamma^\mu \Psi : \partial_\mu u \tag{23}$$

$$= i\partial_\mu (ie : \bar{\Psi}\gamma^\mu \Psi : u) = i\partial_\mu T^\mu_{1/1}(x). \tag{24}$$

Assuming that the operation of commuting with Q commutes with time-ordering, we obtain

$$[Q, T_n(x_1, ...x_n)] = i\sum_{l=1}^{n} \partial_\mu^{x_l} T^\mu_{n/l}(x_1, ...x_n) = (sum\, of\, divergences) \quad , \tag{25}$$

where $T^\mu_{n/l}$ is a mathematically rigorous version of the time-ordered product

$$T^\mu_{n/l}(x_1, ..., x_n)\text{"}=\text{"}\, T(T_1(x_1)...T^\mu_{1/1}(x_l)...T_1(x_n)) \quad , \tag{26}$$

constructed by means of the method of Epstein and Glaser described above. We define (25) to be the condition of gauge invariance [3]. For a fixed x_l we consider from T_n all terms with the external field operator $A_\mu(x_l)$

$$T_n(x_1, ...x_n) =: t_l^\mu(x_1, ...x_n) A_\mu(x_l) : + ... \tag{27}$$

(the dots represent terms without $A_\mu(x_l)$). Then gauge invariance requires

$$\partial_\mu^l [t_l^\mu(x_1,...x_n)u(x_l)] = t_l^\mu(x_1,...x_n)\partial_\mu u(x_l) \tag{28}$$

or

$$\partial_\mu^l t_l^\mu(x_1,...x_n) = 0. \tag{29}$$

It is an interesting observation that although the photon is massive, it is not necessary to introduce a 'Higgs' field to give an explanation for this fact.

3 Unitarity

Eq. (29) is the usual gauge invariance condition as in the massless case [3], where no scalar Φ is needed. Moreover, Φ and the ghost fields do not couple at all. Therefore, we have to explain why the unphysical fields have been introduced. The reason is that it allows to prove unitarity of the S-matrix on the physical Hilbert space H_{phys}, which is a subspace of the Fock-Hilbert space F containing also the unphysical ghosts and scalars.

The basic property for unitarity is the nilpotency of the gauge charge Q

$$Q^2 = \frac{1}{2}\{Q, Q\} = 0, \tag{30}$$

and the Krein structure on the Fock-Hilbert space [5,6,7,8]. Then the physical Fock space can be expressed by the kernel and the range of Q, namely

$$H_{phys} = ker\, Q \ominus ran\, Q = ker\{Q, Q^+\}. \tag{31}$$

This can be most easily seen by realizing the various field operators on a *positive definite* Fock-Hilbert space F:

$$A^\mu(x) = (2\pi)^{-3/2} \sum_{\lambda=0}^{3} \int \frac{d^3k}{\sqrt{2\omega}} \Big(\epsilon_\lambda^\mu(\mathbf{k}) a_\lambda(\mathbf{k}) e^{-ikx} \pm (\epsilon_\lambda^\mu(\mathbf{k}) a_\lambda^+(\mathbf{k}) e^{+ikx}\Big) \quad , \quad \omega = \sqrt{\mathbf{k}^2 + m^2} \tag{32}$$

where ϵ_λ^μ are four polarization vectors satisfying

$$\epsilon_0^\mu \stackrel{def}{=} \frac{k^\mu}{m} \quad , \quad g_{\mu\nu}\epsilon_\lambda^\mu \epsilon_\kappa^\nu = g_{\kappa\lambda} \quad , \tag{33}$$

$$\sum_{\lambda=0}^{3} g_{\lambda\lambda}\epsilon_\lambda^\mu \epsilon_\lambda^\nu = g^{\mu\nu} \quad , \quad \epsilon_\lambda^{\mu *} = \epsilon_\lambda^\mu, \tag{34}$$

and we have a minus sign for $\lambda = 0$ in (32) to be consistent with Lorentz invariance. A similar asymmetry occurs in the ghost sector

$$u(x) = (2\pi)^{-3/2} \int \frac{d^3k}{\sqrt{2\omega}} \Big(c_2(\mathbf{k}) e^{-ikx} + c_1(\mathbf{k})^+ e^{ikx} \Big), \tag{35}$$

$$\tilde{u}(x) = (2\pi)^{-3/2} \int \frac{d^3k}{\sqrt{2\omega}} \Big(-c_1(\mathbf{k}) e^{-ikx} + c_2(\mathbf{k})^+ e^{ikx} \Big). \tag{36}$$

All creation and annihilation operators satisfy the usual commutation relations. Then the proof of unitarity is exactly the same as in [5,6].

We wish to emphasize that we are not forced to represent the gauge potential in the Feynman gauge as in (32). If we would not do so, the unphysical particles would acquire a mass depending on the gauge fixing parameter. Furthermore, in the case of a massless photon, the above considerations remain valid with a little exception: The *unphysical* scalar field Φ would not appear anymore in the gauge charge Q, therefore it would become physical and its mass could be chosen arbitrarily, or the field could be removed from the theory.

The full power of the above concept shows up if non-abelean gauge fields are introduced (e.g. in electroweak theory [9,10]). The example which follows shows some essential features of the more complicated discussion in case of the elektroweak theory. For simplicity, we will demonstrate in the following section how perturbative gauge invariance fixes all couplings in the case of an Abelian theory. In a sense, we will *derive* the 'Higgs'-potential.

4 The Abelian Higgs Model

4.1 Gauge Invariance at First Order

Consider the simple case of classical Abelian U(1) gauge theory [11], given by the Lagrangean

$$\mathcal{L} = (\partial_\mu + igB_\mu)\varphi^+(\partial^\mu - igB^\mu)\varphi + \mu^2\varphi^+\varphi - \lambda(\varphi^+\varphi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{37}$$

$$F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{38}$$

If we assume that the scalar field φ develops a vacuum expectation value $|\langle 0|\varphi|0\rangle| = v/\sqrt{2} = (\mu^2/2\lambda)^{1/2}$, then we obtain in the unitary gauge the Lagrangean

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_H^2\phi^2 - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}m^2 A_\mu A^\mu$$
$$+ g^2 v A_\mu A^\mu \phi + \frac{1}{2}g^2 A_\mu A^\mu \phi^2 - \lambda v \phi^3 - \frac{1}{4}\lambda\phi^4, \tag{39}$$

$$m = gv \quad , \quad m_H = \sqrt{2}\mu \quad , \tag{40}$$

where ϕ is hermititan and $(A_\mu, (v+\phi(x))/\sqrt{2})$ are obtained from (B_μ, φ) by a local $U(1)$ transformation

$$\varphi(x) = \frac{1}{\sqrt{2}}(v+\phi(x))e^{i\xi(x)/v} \quad , \quad B_\mu(x) = A_\mu(x) + \frac{1}{gv}\partial_\mu\xi(x) \tag{41}$$

Now we derive the whole quantum theory in a totally different way. Our starting point is the first order coupling $\sim A_\mu A^\mu \phi$ of the physical fields A_μ and ϕ with masses m and m_H, respectively. Furthermore, we introduce the unphysical scalar field Φ which appears in the gauge charge Q. The latter is still given by (17) and the guiding principle is the operator gauge invariance (25). Then a general ansatz for the first order coupling, containing only trilinear terms in the free fields and leading to a renormalizable theory, is

$$T_1(x) = igm : [A_\mu A^\mu\phi + \alpha A_\mu A^\mu \Phi + \beta_1 u\tilde{u}\phi + \beta_2 u\tilde{u}\Phi + \beta_3 A_\mu u\partial^\mu\tilde{u}$$
$$+ \gamma A_\mu(\phi\partial^\mu\Phi - \Phi\partial^\mu\phi) + \delta_1\Phi^3 + \delta_2\Phi^2\phi + \delta_3\Phi\phi^2 + \delta_4\phi^3] : \tag{42}$$

We calculate $d_Q T_1 = [Q, T_1]$ and obtain

$$d_Q T_1 = -gm : [2\partial_\mu(u(A^\mu\phi + \alpha A^\mu\Phi)) + \gamma\partial_\mu(u(\phi\partial^\mu\Phi - \Phi\partial^\mu\phi))$$
$$+\gamma m\partial_\mu(uA^\mu\phi) - 2u\partial_\mu A^\mu\phi - 2uA^\mu\partial_\mu\phi - 2\alpha u\partial_\mu A^\mu\Phi - 2\alpha uA^\mu\partial_\mu\Phi$$
$$+\alpha m u A_\mu A^\mu + \beta_1 u\partial_\mu A^\mu\phi + \beta_1 m u\Phi\phi + \beta_2 u\partial_\mu A^\mu\Phi + \beta_2 m u\Phi^2$$
$$\beta_3(\partial^\mu u u\partial_\mu\tilde{u} + A^\mu u\partial_\mu(\partial_\nu A^\nu + m\Phi))$$
$$+\gamma m^2 u\phi\Phi - \gamma m_H^2 u\phi\Phi - \gamma m u\partial_\mu A^\mu\phi - 2\gamma m u A_\mu\partial^\mu\phi$$

$$+ 3\delta_1 mu\Phi^2 + 2\delta_2 mu\Phi\phi + \delta_3 mu\phi^2] : \tag{43}$$

where we have taken out the derivatives of the ghost fields. Since $d_Q T_1$ has to be a pure divergence, the terms which are not of this form must cancel. This fixes most of the free parameters. We immediately obtain

$$T_1 = igm : [A^\mu A_\mu \phi + u\tilde{u}\phi - \frac{1}{m} A_\mu (\phi\partial^\mu \Phi - \Phi \partial^\mu \phi) - \frac{m_H^2}{2m^2}\phi\Phi^2 + \delta_4 \phi^3] : \tag{44}$$

and

$$d_Q T_1 = -gm : \partial_\mu [(uA^\mu \phi) - \frac{1}{m} u(\phi\partial^\mu \Phi - \Phi\partial^\mu \phi)] : \stackrel{def}{=} i\partial_\mu T^\mu_{1/1}. \tag{45}$$

Obviously, the quadrilinear couplings in (39) are still missing, and δ_4 is not yet fixed. Therefore, we have to discuss gauge invariance at second and third order.

4.2 Gauge Invariance at Second and Third Order

Following the inductive construction of Epstein and Glaser, we have to calculate first the causal distribution

$$D_2(x,y) = T_1(x)T_1(y) - T_1(y)T_1(x) = -A'_2(x,y) + R'_2(x,y). \tag{46}$$

The main problem is whether gauge invariance can be preserved in the distribution splitting. Obviously, D_2 is gauge invariant:

$$d_Q D_2(x,y) = [d_Q T_1(x), T_1(y)] + [T_1(x), d_Q T_1(y)] =$$

$$i\partial^x_\mu [T^\mu_{1/1}(x), T_1(y)] + i\partial^y_\mu [T_1(x), T^\mu_{1/1}(y)] \stackrel{def}{=} i\partial^x_\mu D^\mu_{2/1}(x,y) + i\partial^y_\mu D^\mu_{2/2}(x,y). \tag{47}$$

Since the retarded part R_2 agrees with D_2 on the forward light cone $V^+ \setminus \{x = y\}$ and similarly for $R^\mu_{2/1}$,$R^\mu_{2/2}$, gauge invariance of R_2 can only be violated by local terms $\sim D^a\delta(x-y)$. But such local terms are precisely the freedom of normalization in the distribution splitting. If the normalization terms N_2,$N^\mu_{2/1}$, $N^\mu_{2/2}$ can be chosen in such a way that

$$d_Q(R_2 + N_2) = i\partial^x_\mu (R^\mu_{2/1} + N^\mu_{2/1}) + i\partial^y_\mu (R^\mu_{2/2} + N^\mu_{2/2}) \tag{48}$$

holds, then the theory is gauge invariant in second order. Note that the distribution $T_2 = R_2 + N_2 - R_2'$ then fulfils (48), too. The local terms on the right-hand side of (48), which come from the causal splitting, are called anomalies. The ordinary axial anomalies in the standard model are of the same kind, they appear in the third order triangle diagrams with axial vector couplings to fermions [10].

We consider the following example: In the commutator $[T^{\mu}_{1/1}(x), T_1(y)]$ appears the term

$$-g^2 m : u(x)\Phi(x)[\partial_\mu \phi(x), \phi(y)] A_\nu(y) A^\nu(y) := i g^2 m : u(x)\Phi(x) A_\nu(y) A^\nu(y) : \partial^\mu D_{m_H}(x-y). \tag{49}$$

After splitting this causal distribution the Pauli-Jordan distribution D_{m_H} is replaced by the retarded distribution $D^{ret}_{m_H}$. If we calculate now the divergence of (49), we get an anomaly

$$\frac{A_1}{2} = i g^2 m : u\Phi A_\nu A^\nu : \delta(x-y), \tag{50}$$

because

$$\partial^x_\mu \partial^\mu_x D^{ret}_m(x-y) = -m^2 D^{ret}_m(x-y) + \delta(x-y). \tag{51}$$

The terms with x and y interchanged lead to the same contribution. But in the causal distribution $D_2 = [T_1(x), T_1(y)]$ appears the term

$$-g^2 : A_\mu(x)\Phi(x)[\partial^\mu \phi(x), \partial^\nu \phi(y)] A_\nu(y)\Phi(y) : \tag{52}$$

$$= -i g^2 : A_\mu(x) A_\nu(y) \Phi(x)\Phi(y) : \partial^\mu_x \partial^\nu_x D_{m_H}(x-y) \tag{53}$$

which has singular degree $\omega = 0$ [2,3] and therefore allows a normalization term in the splitted distribution

$$\partial^\nu_x \partial^\mu_x D^{ret}(x-y) \to \partial^\nu_x \partial^\mu_x D^{ret}(x-y) + C g^{\mu\nu} \delta(x-y). \tag{54}$$

Since

$$d_Q(: \Phi^2 A_\mu A^\mu : C\delta(x-y)) = 2iCm : u\Phi A_\mu A^\mu : \delta(x-y) + ... \tag{55}$$

we can compensate the anomaly (50) by choosing $C = ig^2$. In this way we obtain the quadrilinear couplings of the theory as normalization terms in higher orders. We give here the complete list of all normalization terms for tree diagrams in second order:

$$N_1 = ig^2 : A_\mu A^\mu \Phi^2 : \delta(x-y) \tag{56}$$

$$N_2 = ig^2 : A_\mu A^\mu \phi^2 : \delta(x-y) \tag{57}$$

$$N_3 = -ig^2 \frac{m_H^2}{4m^2} : \Phi^4 : \delta(x-y) \tag{58}$$

$$N_4 = ig^2 (\frac{m_H^2}{m^2} + 3\delta_4) : \phi^2 \Phi^2 : \delta(x-y) \tag{59}$$

$$N_5 = ig^2 \lambda' : \phi^4 : \delta(x-y) \quad , \quad \lambda' \quad \text{still free} \tag{60}$$

The remaining free parameters δ_4 and λ' can be determined by considering the anomalies $\sim \delta(x-z)\delta(y-z)$ of tree diagrams in third order. They arise in the splitting of terms in

$$D^\mu_{3/1}(x,y,z) = [T^\mu_{1/1}(x), T_2(y,z)] + ... \tag{61}$$

where $T^\mu_{1/1}$ (45) gets contracted with a normalization term N_{1-5} in T_2. Considering all anomalies $\sim: u\Phi\phi^3 :$, gauge invariance requires

$$2\lambda' = \frac{m_H^2}{m^2} + 3\delta_4, \tag{62}$$

and from the anomalies $\sim: u\phi\Phi^3 :$ we obtain

$$\delta_4 = -\frac{m_H^2}{2m^2}, \tag{63}$$

in agreement with (39).

Besides some basic assumptions concerning simplicity (42), we have constructed the theory with the help of a guiding principle, namely perturbative quantum gauge invariance, which, after construction, is manifest in our approach.

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